

AceBear Security Contest 2019

Challenge: \cotan

Hi everyone,

Welcome to the \cotan challenge, I hope you still remember this formula:

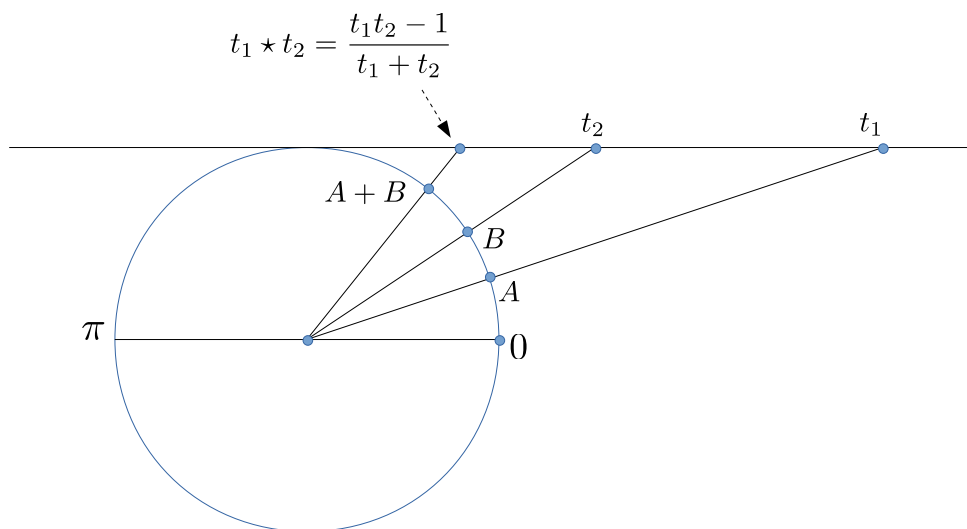
$$\cot(A + B) = \frac{\cot(A)\cot(B) - 1}{\cot(A) + \cot(B)} \quad (1)$$

Let's define a new operation, \star , for which $t_1 \star t_2 = \frac{t_1 t_2 - 1}{t_1 + t_2}$, then we can rewrite (1) as

$$\cot(A + B) = \cot(A) \star \cot(B) \quad (2)$$

Let $+_\pi$ denote addition modulo π , then this subset of \mathbb{R} : $[0, \pi)$ (including 0 but not π) under $+_\pi$ is clearly a group (in fact, it is the quotient group $\frac{\mathbb{R}}{2\pi\mathbb{R}}$).

Now, it's time to introduce another group. Let f be a mapping which maps $x \in [0, \pi)$ to $\cot(x) \in \mathbb{R} \cup \{\infty\}$, then f carries the group structure of $[0, \pi)$ under $+_\pi$ to $\mathbb{R} \cup \{\infty\}$ under \star (see the figure below).



And here's the challenge for you. We will be working on F_p instead of \mathbb{R} . I give you $\underbrace{2 \star 2 \star 2 \star \dots 2}_{k \text{ times}}$ and you need to find k , that's all. Good luck!